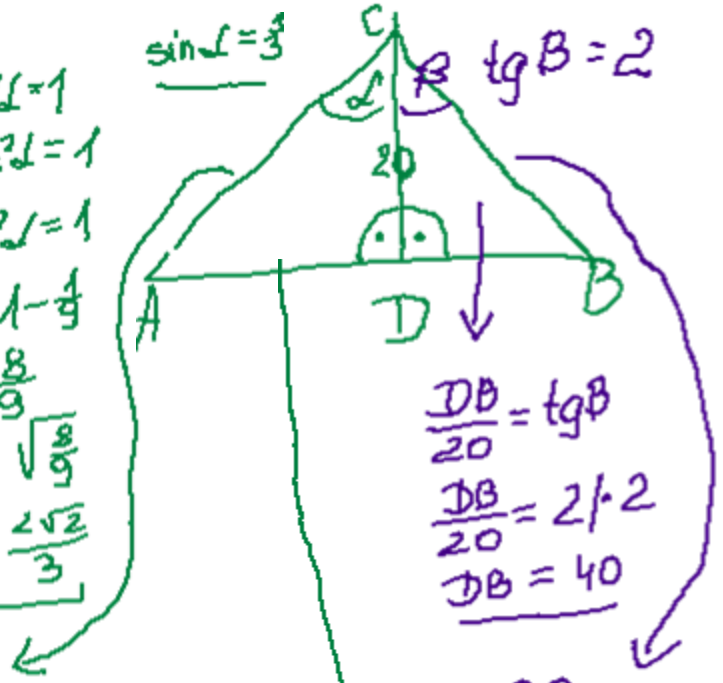
Wysokość CD trójkąta ABC ma długość 20 i tworzy z bokiem AC kąt alfa, taki, że sin alfa=1/3, a z bokiem BC kąt beta taki,ze tg beta=2. Wyznacz obwód trójkąta.



$\sin\alpha = \frac{1}{3}$

$\sin^2\alpha + \cos^2\alpha = 1$

$(\frac{1}{3})^2 + \cos^2\alpha = 1$

$\frac{1}{9} + \cos^2\alpha = 1$

$\cos^2\alpha = 1 - \frac{1}{9}$

$\cos^2\alpha = \frac{8}{9}$

$\cos\alpha = \sqrt{\frac{8}{9}}$

$\cos\alpha = \frac{2\sqrt{2}}{3}$

$\frac{20}{AC} = \cos\alpha$

$\frac{20}{AC} = \frac{2\sqrt{2}}{3}$

$2\sqrt{2}AC = 60 \;/:2\sqrt{2}$

$AC = \frac{30}{\sqrt{2}}$

$AC = \frac{30}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$AC = \frac{30\sqrt{2}}{2}$

$AC = 15\sqrt{2}$

$\frac{AD}{AC} = \sin\alpha$

$\frac{AD}{15\sqrt{2}} = \frac{1}{3}$

$AD = \frac{15\sqrt{2}}{3}$

$AD = 5\sqrt{2}$

$tg\beta = 2$

$tg\beta = \frac{\sin\beta}{\cos\beta} = 2$

$\sin\beta = 2\cos\beta$

$\sin^2\beta + \cos^2\beta = 1$

$(2\cos\beta)^2 + \cos^2\beta = 1$

$4\cos^2\beta + \cos^2\beta = 1$

$5\cos^2\beta = 1 \;/:5$

$\cos^2\beta = \frac{1}{5}$

$\cos\beta = \frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$

$\cos\beta = \frac{\sqrt{5}}{5}$

$\frac{DB}{20} = tg\beta$

$\frac{DB}{20} = 2 \;/\cdot 2$

$DB = 40$

$\frac{20}{CB} = \cos\beta$

$\frac{20}{CB} = \frac{\sqrt{5}}{5}$

$\sqrt{5}\,CB = 5 \cdot 20$

$\sqrt{5}\,CB = 100 \;/:\sqrt{5}$

$CB = \frac{100}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$

$CB = \frac{100\sqrt{5}}{5}$

$CB = 20\sqrt{5}$

$Ob = AD + DB + BC + CA$

$Ob = 5\sqrt{2} + 40 + 20\sqrt{5} + 15\sqrt{2}$

$Ob = 20\sqrt{2} + 20\sqrt{5} + 40$

$Ob = 20(\sqrt{2} + \sqrt{5} + 2)$

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